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SOLUTION OF PROB. 174 BY CHAS. H. KUMMELL. (CONTINUED FROM P. 157.)

But $(\Gamma(\tfrac{1}{12}n)$ may now be likewise determined. We have by (4), p. 120,

$$\Gamma(\tfrac{1}{4}) = \frac{(2\pi)^{3/4}}{I^{1/2}} = 2^{3/4}\pi^{3/4}\left\Vert \tfrac{1}{2}\left\{ \sqrt{2}\overset{+}{\underset{\times}{}}1 \right\}^{\frac{1}{2}}\right\Vert^{-\frac{1}{2}}. \tag{11}$$

By theorem (2) we have

$$\Gamma(\tfrac{11}{12}) = \frac{\pi}{\sin\tfrac{1}{12}\pi\ \Gamma(\tfrac{1}{12})}, \tag{12}$$

$$\Gamma(\tfrac{9}{12}) = \Gamma(\tfrac{3}{4}) = \frac{\pi}{\sin\tfrac{1}{4}\pi\ \Gamma(\tfrac{1}{4})} = \frac{2^{1/2}\pi}{\Gamma(\tfrac{1}{4})}, \tag{13}$$

$$\Gamma(\tfrac{7}{12}) = \frac{\pi}{\sin\tfrac{5}{12}\pi\ \Gamma(\tfrac{5}{12})} = \frac{\pi}{\cos\tfrac{1}{12}\pi\ \Gamma(\tfrac{5}{12})}. \tag{14}$$

$\Gamma(\tfrac{1}{4})$ being given by (11) only $\Gamma(\tfrac{1}{12})$ and $\Gamma(\tfrac{5}{12})$ must yet be determined. Placing in (5) $n = \tfrac{1}{12}$ and $r = 2$ we have

$$\Gamma(\tfrac{1}{12})\,\Gamma(\tfrac{7}{12}) = (2\pi)^{1/2}2^{1/3}\Gamma(\tfrac{1}{6}),$$

and dividing by (17) $\quad \Gamma(\tfrac{1}{12}) = 2^{\frac{5}{6}}\,\pi^{-1/2}\cos\tfrac{1}{12}\pi\,\Gamma(\tfrac{1}{6})\,\Gamma(\tfrac{5}{12})$

$$= 2^{1/2}3^{1/2}\pi^{-1}\cos\tfrac{1}{12}\pi\,\Gamma(\tfrac{1}{3})^2\Gamma(\tfrac{5}{12}) \text{ by (6)}. \tag{15}$$

Also placing in (5) $n = \tfrac{1}{12}$ and $r = 3$, then

$$\Gamma(\tfrac{1}{12})\,\Gamma(\tfrac{5}{12})\,\Gamma(\tfrac{3}{4}) = 2.3^{1/4}\pi\,\Gamma(\tfrac{1}{4}),$$

and dividing by (16) $\quad \Gamma(\tfrac{1}{12})\,\Gamma(\tfrac{5}{12}) = 2^{1/2}3^{1/4}\,\Gamma(\tfrac{1}{4})^2. \qquad (16)$

Multiplying (16) by (15) and extracting the square root we obtain

$$\Gamma(\tfrac{1}{12}) = 2^{\frac{1}{2}}3^{\frac{3}{8}}\pi^{-\frac{1}{2}}(\cos\tfrac{1}{12}\pi)^{\frac{1}{2}}\,\Gamma(\tfrac{1}{3})\,\Gamma(\tfrac{1}{4}). \tag{17}$$

Also dividing (16) by (17) we obtain

$$\Gamma(\tfrac{5}{12}) = 3^{-\frac{1}{8}}\pi^{\frac{1}{2}}\left(\cos\frac{\pi}{12}\right)^{-\frac{1}{2}}\frac{\Gamma(\tfrac{1}{4})}{\Gamma(\tfrac{1}{3})}. \tag{18}$$

This together with (1), (3), (4), (6), (10), (11), (12), (13), (14), (17) determines $\Gamma(\tfrac{1}{12}n)$.

The general problem to determine $\Gamma(n)$ may now be solved with that degree of accuracy usually required. It is known now at intervals of $\tfrac{1}{12}$ and if we can determine it between r and $r + \tfrac{1}{2}$ at smaller intervals we know $\Gamma(n)$ in its whole extent.

We have then by Lagrange's interpolation formula

$$\Gamma(r+n) = 12n.\frac{12n-1}{1}\cdot\frac{12n-2}{2}\cdots\frac{12n-6}{6}\Big[\frac{1}{12n}\Gamma(r) - \frac{6}{12n-1}\Gamma(r+\tfrac{1}{12})$$

$$+\frac{15}{12n-2}\Gamma(r+\tfrac{1}{6}) - \frac{20}{12n-3}\Gamma(r+\tfrac{1}{4}) + \frac{15}{12n-4}\Gamma(r+\tfrac{1}{3}) - \frac{6}{12n-5}\Gamma(r+\tfrac{5}{12})$$

$$+\frac{1}{12n--6}\Gamma(r+\tfrac{1}{2})\Big]. \tag{19}$$

Since $\Gamma(n)$ has its minimum value between $n=1$ and $n=2$ it is advantageous to place $r=1$, hence

$$\Gamma(1+n)=12n.\frac{12n-1}{1}.\frac{12n-2}{2}\cdots\frac{12n-6}{6}\Bigg[\frac{1}{12n}-\frac{6}{12n-1}\Gamma(\tfrac{13}{12})$$
$$+\frac{15}{12n-2}\Gamma(\tfrac{7}{6})-\frac{20}{12n-3}\Gamma(\tfrac{5}{4})+\frac{15}{12n-4}\Gamma(\tfrac{4}{3})-\frac{6}{12n-5}\Gamma(\tfrac{17}{12})$$
$$+\frac{1}{12n-6}\Gamma(\tfrac{3}{2})\Bigg], \qquad (20)$$

n being some fraction $< \frac{1}{2}$.

The following table of the gamma function was computed by the above convenient formula:

n.	$\log \Gamma(n)$.
1.000	0.0000000
1.083	9.9814951
1.166	9.9674166
1.250	9.9573211
1.333	9.9508415
1.416	9.9476700
1.500	9.9475450

and these values being used in (20) for values of $n < \frac{1}{2}$ and > 0 will give results correct to sixth differences.

NOTE ON THE METHOD OF LEAST SQUARES.

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WHEN a greater number of points are given or observed than are sufficient to determine any point, line or surface, that point, line or surface which makes the sum of the squares of the errors of situation a minimum, has the greatest probability, and is therefore the one determined by these points.

(1). Let the coordinates, (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_n, z_n) of n points, be given or measured, and represent by d_1, d_2 . . . d_n, the distances respectively from the n points to any point (α, β, γ).

If $m =$ the number of points on a unit of surface, then the probability that a point, taken at random on any of the surfaces of the spheres whose centers are (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_n, z_n), and radii respectively $d_1, d_2, \ldots d_n$, shall be at the point (α, β, γ), is $\frac{1}{\pi m(d_1^2+d_2^2+\ldots+d_n^2)}$, which probability is greatest when $d_1^2+d_2^2+\ldots+d_n^2 =$ a minimum.